TEACHING STRATEGIES FOR MENTAL MATHEMATICS (FOUNDATION PHASE)

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Being able to do calculations in your head is an important life skill and an important part of mathematics. Mental mathematics is also a very important component of the NCS Curriculum and Assessment Policy for Mathematics. The CAPS document lists the number bonds and multiplication table facts that Foundation Phase learners are expected to know and recall for each grade. However, to improve mental calculations, learners need to be taught the most efficient strategies explicitly. This workshop aims to revise the strategies for calculation suggested in the CAPS document and propose some activities that can be used to practise these strategies.

RAPID RECALL CALCULATION FACTS

Tables of the calculation facts for each grade are given below. The list is not exhaustive but includes the key strategies that can be used when developing mental mathematics skills and numeracy. Each year is based on the previous year and teachers should ensure that the knowledge from previous years is revised and carried forward.

This table has been based on the NCS Curriculum and Assessment Policy Statement for Foundation Phase Mathematics.

Grade 1

Grade 1 learners should have	Mental Strategies
Number concept: range 20 • Order a given set of selected numbers. • Compare numbers up to 20 and say which is and more or less	**Number concept: range 20** • Order a given set of selected numbers. • Compare numbers up to 20 and say which is and more or less
Rapid recall:	**Use calculation strategies to add and subtract efficiently:**
Number bonds to 10	Put the larger number first in order to count on or count back
Addition and subtraction facts to 10	Number line
	Doubling and halving
	Building up and breaking down

Table 1: Summary of Grade 1 mental mathematics facts and mental strategies

Grade 2

Grade 2 learners should have	Mental Strategies
Number concept: range 99 • Order a given set of selected numbers. • Compare numbers up to 99 and say which is and more or less	**Number concept: range 99** • Order a given set of selected numbers. • Compare numbers up to 99 and say which is and more or less
Rapid recall:	**Use calculation strategies to add and subtract efficiently:**
Addition and subtraction facts to 20	Put the larger number first in order to count on or count back
Add or subtract multiples of 10 from 0 to 100	Use the relationship between addition and subtraction
	Number line
	Doubling and halving
	Building up and breaking down

Table 2: Summary of Grade 2 mental mathematics facts and mental strategies

Grade 3

Grade 3 learners should have	Mental Strategies
Number concept: range 999 • Order a given set of selected numbers. • Compare numbers up to 1 000 and say which is and more or less	**Number concept: range 1 000** • Order a given set of selected numbers. • Compare numbers up to 1 000 and say which is and more or less
Rapid recall:	**Use calculation strategies to add and subtract efficiently:**
Addition and subtraction facts to 20	Put the larger number first in order to count on or count back
Add or subtract multiples of 10 from 0 to 100	Number line
Multiplication and division facts for the : * two times table up to 2×10 * ten times table up to 10×10	Doubling and halving
	Building up and breaking down
	Use the relationship between addition and subtraction
	Use the relationship between multiplication and division

Table 3: Summary of Grade 3 mental mathematics facts and mental strategies

CHOOSING STRATEGIES

Ten minutes of mental mathematics is recommended every day. This can involve asking learners quick mental starters like: the number before 7 is ...; two more or less than 18 is ....; and 7 + 2; 8 + 2; 9 + 2, etc. These mental mathematics activities can also take the form of printed exercises where learners work independently and write their answers. Peer assessment should be used to mark the answers. Teachers should call the answers out clearly and slowly, write them on the board or display then on a projector.

The teacher should then select two or three learners to explain the strategy that they used to find the answer(s). Discuss the relative merits of different methods with learners. It is important that a safe environment is created in the class so that learners feel confident to discuss their methods. Questions that can be asked include:

- How did you get that answer?
- Is there another way that you could have found the answer?
- Did anyone find the answer in a different way?
- Can you write down the method in a number sentence?

It is important to emphasize that

- learners need efficient and quick methods for mental mathematics
- learners should choose a method that is sensible
- teachers consolidate the main features of the strategies used at the end of the session.

The mental mathematics programme should be developed systematically over the year. As learners cover topics and develop strategies in the main part of the lesson, they can practise them in the mental mathematics programme.

KEY STRATEGIES AND ACTIVITIES

Although a large part of mental mathematics in the Foundation Phase involves the rapid recall of number facts, it is also important to develop these facts to other similar numbers. For example, if a learner knows that 8 + 8 = 16, then the following calculations can be developed from this fact like:

7 + 8; 7 + 18; 8 + 18 and 18 + 18

Different learners will carry out these calculations using different strategies. It is useful that teachers recognise the different strategies that may be forthcoming. This is in turn can be used to build up a range of different strategies and assist learners to choose more efficient strategies where necessary.

Teachers should encourage learners to

- investigate other strategies
- practice other strategies and so build up confidence in using them
- develop their methods that work efficiently.

The NCS CAPS document recommends that it is useful to do mental mathematics with apparatus and to record what is done. The recommended apparatus includes:

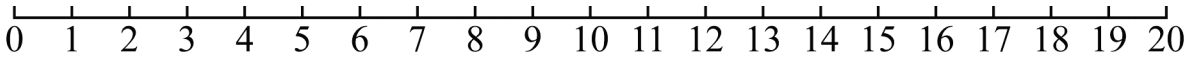
- a number line (structured or empty)
- a number grid
- place value cards (Flard cards)
- counting beads

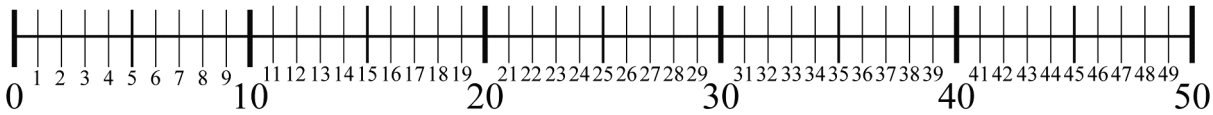
Paper and pen/pencil can also be used to either write the answers, jot down reminders of patterns or draw images that show how the calculation is being performed.

Once learners have mastered a strategy, they should be encouraged to practise it often enough to build up some speed when answering. Teachers can reinforce a particular strategy and then let learners practise it before the mental mathematics test. They should encourage learners to discuss alternate strategies in these sessions.

Counting on and back

A large number line in class allows learners to appreciate the concept of counting forward and backwards more easily. Remind learners that their rulers are also number lines and can be used for counting.





Counting helps learners develop strategies for calculating. It is also useful for recognising patterns. Learners start counting in ones and then later learn to count in twos, fives, tens and so on. When learners 'skip count', they are actually calculating already.

Learners also need to be able to express numbers verbally, in written number symbols as well as written number words.

Using a number line, learners will also appreciate that when adding two numbers together it is easier to count on from the larger number. This method will eventually be replaced by more efficient methods. When subtracting, learners should count back from the larger number (which will be first).

Example 1: 3 + 6

Think	Do
Start with the largest number	6 + 3
Count on to 9	6 + 1 + 1 + 1 = 9

Example 2: 7 – 2

Think	Do
Start with the largest number	7 – 2
Count back from 7 to 5	7 – 1 – 1 = 5

Developing this strategy further, ask learners to count on or back in twos when the smaller number is an even number, for example 16 – 6 . You can also ask learners to count on in tens when the numbers are multiples of ten, for example for 30 + 60, learners count on in tens from 60 to 90.

Activity 1

Write the biggest number first. Then count on.

1 5 + 17 = □ + Δ = ___

2 1 + 9 = □ + Δ = ___

3 3 + 32 = □ + Δ = ___

4 38 + 6 = □ + Δ = ___

5 18 + 3 = = □ + Δ = ___

6 $28 + 5 = \square + \Delta =$ ___

7 $12 + 17 = \square + \Delta =$ ___

8 $14 + 13 = \square + \Delta =$ ___

9 $38 + 9 = \square + \Delta =$ ___

10 $47 + 19 = \square + \Delta =$ ___

Ordering a given set of numbers

Numbers can be added in any order, so $2 + 5 + 8 = 2 + 8 + 5$. When there are three numbers that need to be added together, two will be added first and the answer then added to the third number. Teach learners to look for pairs of numbers that make 10 and add them first. They should all know the number bonds so they would be looking for $1 + 9$; $2 + 8$; $3 + 7$; $4 + 6$ and $5 + 5$.

For example, $2 + 5 + 8 = (2 + 8) + 5 = 10 + 5 = 15$

It is important that learners realise that they can change the order of the numbers when there are just addition signs, but it gets more complicated when there are minus signs. Order does matter in subtraction!

$9 – 7$ does not equal $7 – 9$.

When there is more than one subtraction, the order can be changed. Teachers should judge the confidence of their group before introducing this technique.

$17 – 5 – 8 = 17 – 8 – 5$

Activity 2

Look for pairs of numbers that make 10. Add these first.

1 $3 + 2 + 8 = (2 + \square) + 3 = 10 + \Delta =$ ___

2 $7 + 4 + 3 = (7 + \square) + 4 = \Delta + 4 =$ ___

3 $9 + 8 + 2 = 9 + (8 + 2) = 9 + \square =$ ___

4 $5 + 5 + 9 = (\square + \Delta) + 9 = 10 + 9 =$ ___

5 $2 + 11 + 8 = 11 + (\square + \Delta) =$ ___

6 $4 + 7 + 3 = \square + (\Delta + 3) =$ ___

7 $2 + 3 + 18 = (\square + \Delta) + 3 =$ ___

8 $19 + 5 + 2 = (\square + \Delta) + \triangle = \_\_\_$

9 $27 + 4 + 3 = (\square + \Delta) + \triangle = \_\_\_$

Note:

- Changing the order of the numbers is a strategy that can only be used if the question is written.
- The examples ask learners to find pairs of numbers that make 10, but other 'easy' numbers like 20 or any other multiple of 10 can be used.

Doubling and halving

Practice doubles up to 20 with learners often so that they attain instant recall of them. Learners often find the doubles the easiest facts to remember. They can be used to:

- simplify calculations
- double one number and halve the other in a product.

Activity 3

Complete.

1 $9 + 9$ is double___

2 $8 + 8 = 8 \times$ ___

3 Half of 14 is ___

4 Half of ___ is 10.

5 $7 +$ ___ is double 7.

6 Half of 18 is ___.

7 10×2 is _____ 10

8 Half of ___ is 50.

Near doubles

Doubles can then be used to add numbers that are close to doubles. For example, learners know that $8 + 8 = 16$, so they can be encouraged to see that $8 + 9$ will be one more than 16.

As learners become more confident, give them numbers that are two and then three apart to use to practice this strategy.

Activity 4

1 3 + 4 = (3 + 3) + 1 = ___

2 6 + 5 = (6 + 6) – 1 ___ = ___

3 9 + 10 = (9 + 9) + 1 = ___

4 9 + 8 = (9 + 9) – ___ = ___

5 10 + 11 = (10 + ___) + 1 = ___

Activity 5

	Number	Double	Double + 1	Double + 2
1	4	4 + 4 = 8	4 + 5 = 9	4 + 6 = 10
2	2	2 + 2 = ___	2 + __ = ___	2 + 4 = ___
3	5			
4	15			
5	11			

Change a number to 10 and then subtract or add 1

This strategy is useful for adding numbers that are close to a multiple of 10. When numbers are close to 10 or a multiple of 10, the number to be added can be broken into a multiple of 10 plus a small number or a multiple of 10 minus a small number.

Example: 8 + 9

Think	**Do**
9 can be written as 10 – 1	8 + 10 – 1
Add 10 to 8 and then subtract 1	18 – 1 = 17

Example: 6 + 11

Think	Do
11 can be written as 10 + 1	6 + 10 + 1
Add 10 to 6 and then add 1	16 + 1 = 17

Activity 6

Complete.

1 6 + 9 = 6 + 10 – 1 = ___

2 8 + 9 = 8 + 10 + ___ = ___

3 11 + 7 = 10 + 7 – 1 = ___

4 14 + 9 = 14 + 10 – ___ = ___

5 21 + 7 = 20 + 1 + 7 = ___

Building up and breaking down numbers

Revise place value using Flard cards or calculators. Learners should be able to break down numbers like 247 = 200 + 40 + 7. This is a useful strategy for adding and subtracting. Both numbers can be broken down like this, but it may be quicker to just break one of the numbers.

Example: 8 + 12

Think	Do
Leave the first number and break down the second number.	8 + (10 + 2)
Add 8 + 10 and count on by 2	18 + 2 = 20

Example: 48 – 30

Think	Do
Break down the number that has tens and units	40 + 8 – 30
Rearrange.	40 – 30 + 8
Subtract 30 and count on 8	10 + 8 = 18

Use the relationship between addition and subtraction

Every addition calculation can be replaced by an equivalent subtraction calculation and similarly every subtraction can be replaced by an addition. For example with addition,

$$7 + 11 = 18$$

$$7 = 18 - 11$$

$$11 = 18 - 7$$

For subtraction,

$$20 - 6 = 14$$

$$20 = 14 + 6$$

$$6 = 20 - 14$$

Activity 7

Complete.

1 $12 + \square = 18$ and $18 - \square = 12$

2 $10 - 0 = \square$ and $\square + 0 = 10$

3 $19 - \square = 10$ and $19 - 10 = \square$

4 $\square + 12 = 15$ and $15 - \square = 12$

5 $14 - 9 = \square$ and $14 - \square = 9$

6 $6 + \square = 20$ and $20 - 6 = \square$

7 $15 + \square = 19$ and $19 - 15 = \square$

8 $9 - 4 = \square$ and $\square - 5 = 4$

9 $9 + \square = 16$ and $16 - 7 = \square$

10 $19 + 17 = \square$ and $\square - 19 = 16$

Use relationship between multiplication and division

Grouping actual physical objects and numbers is essential for multiplication and division. Multiplication is usually where objects are grouped together and is associated with repeated addition and doubling. Division is associated with breaking up a number of objects into equal groupings, repeated subtraction and halving. Teachers should give learners a lot of activities that involve forward and backward skip counting.

Division and multiplication are inverse operations. Any two whole numbers can be multiplied to get another whole number, but this is not the same for division. It is a very useful strategy for learners to know which of the numbers will result in a whole number when dividing.

Grade 3 learners must be able to rapidly recall multiplication and division facts for the two and ten times tables.

This is a gradual process and learners in Grade 1 and 2 can be encouraged to start by doing the following exercises:

1 Skip count in twos

2 Skip count in tens

Grade 3 learners should:

1 recall the two times table up to 2×10

2 recall the ten times table up to 10×10

3 recall division facts for the two and 10 times table.

For every multiplication there is a division sum and vice versa.

$$7 \times 10 = 70$$

$$7 = 70 \div 10$$

$$10 = 70 \div 7$$

For division,

$$20 \div 2 = 10$$

$$20 = 10 \times 2$$

$$2 = 20 \div 10$$

Activity 8

Complete the empty blocks.

		× 2	×10	÷2	double	half	÷10
1	4						
2	10						
3	20						
4	14						
5	7						
6	17						
7	19						

ESTIMATION

Encourage estimations and checking answers throughout the year.

Estimating is the ability to make reasonable guesses about a quantity. In the lower grades, learners deal with estimations informally and do not learn to round off. It is however important that these younger learners gain experience with estimation and comparing whether their estimate is larger or smaller than the actual count. They need to be able to look at a group of up to 20 objects and have a good sense of whether there are about 5, 10, 15 or 20 objects.

CONCLUSION

The strategies listed above should be practised throughout the year in a structured mental mathematics programme. Learners can also play mathematical games to practise and memorise number facts. Calculators can be used to enhance the understanding of the strategies, but they should not be used during mental mathematics tests.

Mental mathematics is one of the most important tools for learning mathematics. It not only means to calculate quickly, but involves conceptual understanding and problem solving.